

NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CALIFORNIA



THESIS

REDUCTION OF A LARGE-SCALE GLOBAL  
MOBILITY OPTIMIZATION MODEL  
THROUGH AGGREGATION

by

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December 1996

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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE December 1996	3. REPORT TYPE AND DATES COVERED Master's Thesis		
4. TITLE AND SUBTITLE REDUCTION OF A LARGE-SCALE GLOBAL MOBILITY OPTIMIZATION MODEL THROUGH AGGREGATION		5. FUNDING NUMBERS		
6. AUTHOR(S) Fuller, David F. II		8. PERFORMING ORGANIZATION REPORT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Air Force Studies and Analysis Agency		11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.		
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.		12b. DISTRIBUTION CODE		
13. ABSTRACT (maximum 200 words)  This thesis develops a two-stage aggregation/ disaggregation model based on THRUPUT2 (Morton, Rosenthal and Lim, 1995), a mobility optimization model used to analyze the ability of the Armed Forces of the United States to conduct airlifts in support of major military operations. For a given fleet of aircraft, a given network of routes, and a given set of unit movement requirements over time, THRUPUT2 schedules airlift to minimize late deliveries and non-deliveries. The linear programming model presented is based on THRUPUT2, but aggregates those units which share the same origin-destination pair and have overlapping time periods and therefore creates a smaller linear program. This reduction in size will consequently decrease the time needed to solve, which is desirable because repeated runs of this model are necessary to generate analytic insight and develop recommendations. The thesis further develops a disaggregation model which will remove the aggregations of the first, and therefore offer resolution similar to that of THRUPUT2.				
14. SUBJECT TERMS Optimization Modeling, Linear Programming, Aggregation, Model Reduction, Airlift			15. NUMBER OF PAGES 70	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	



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OPTIMIZATION MODEL THROUGH AGGREGATION**

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Submitted in partial fulfillment  
of the requirements for the degrees of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

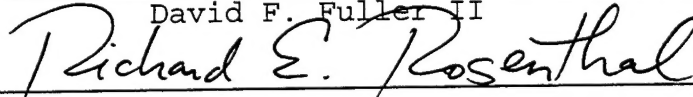
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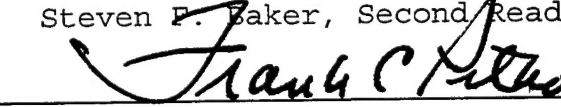


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## ABSTRACT

This thesis develops a two-stage aggregation/disaggregation model based on THRUPUT2 (Morton, Rosenthal and Lim, 1995), a mobility optimization model used to analyze the ability of the Armed Forces of the United States to conduct airlift in support of major military operations. For a given fleet of aircraft, a given network of routes, and a given set of unit movement requirements over time, THRUPUT2 schedules airlift to minimize late deliveries and non-deliveries subject to physical and policy constraints. The linear programming model presented is based on THRUPUT2, but aggregates those units which share the same origin-destination pair and have overlapping time periods, thereby creating a smaller linear program. This reduction in size will consequently decrease the time needed to solve, which is desirable because repeated runs of this model are necessary to generate analytic insight and develop recommendations. The thesis further develops a disaggregation model which will remove the aggregations of the first, and therefore offer resolution similar to that of THRUPUT2.



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## ACKNOWLEDGMENTS

I would like to acknowledge the contribution of the U.S. Air Force Studies and Analysis Agency. Without their funding, neither my research nor that which preceeded it would have been possible.

I would also like to express my tremendous appreciation to the following people whose efforts have helped me in the completion of this thesis.

- Professor Richard E. Rosenthal for allowing me to work on his Air Force Mobility Project, for his exceptional teaching, for his patience, and for his willingness to allow me to explore the problem.
- To Major Steve Baker who provided tremendous insight, a ready ear, and assistance from start to finish.
- To Major Kirk Yost for listening to my ideas and being willing to tell me whether or not they were crazy.

Finally, I would like to thank my lovely wife Marti, for her patience with me, her love, and her editing. I owe her a great deal of gratitude and a honeymoon.



## EXECUTIVE SUMMARY

This thesis develops a two-stage aggregation/disaggregation model based on THRUPUT2 (Morton, Rosenthal and Lim, 1995), a mobility optimization model developed at the Naval Postgraduate School for the Air Force Studies and Analysis Agency (AFSAA). Throughput II is used to analyze the ability of the Armed Forces of the United States to conduct airlift in support of major military operations. For a given fleet of aircraft, a given network of routes, and a given set of movement requirements over time, THRUPUT2 schedules airlift to minimize late deliveries and non-deliveries subject to physical and policy constraints.

The first of the two models developed is the Unit Aggregation Model. This model is based on THRUPUT2, but aggregates those units which share the same origin-destination pair and have overlapping time periods. This aggregation reduces the size of the model. This reduction in size will consequently decrease the time needed to solve the model. Because repeated runs of this model are necessary to generate analytic insight and develop recommendations, this time savings can be very helpful. However, this time savings is not without cost. The aggregation causes a loss of model fidelity at the unit level. This loss of resolution can be partially recovered through the use of a disaggregation model.

The disaggregation model removes the aggregations of the Unit Aggregation Model. It achieves this goal by

assigning unit equipment and troops to those delivery missions scheduled by the Unit Aggregation Model. The resolution is therefore similar to that of THRUPUT2.

The Unit Aggregation Model's size is reduced to about 10% that of THRUPUT2's size for a realistic scenario involving two major regional contingencies. For this scenario, the aggregation/disaggregation model combination takes twenty minutes to generate and solve using GAMS/OSL software on an IBM RS6000/590 computer. THRUPUT2 takes about three hours to generate and solve with the same computer and software.

## **I. INTRODUCTION**

### **A. BACKGROUND OF THRUPUT2**

Operations Desert Shield and Desert Storm revealed deficiencies in tools for planning and analyzing air mobility during Major Regional Contingencies (MRCs). After these operations, the United States Air Force Studies and Analysis Agency (USAF/SAA) began examining the use of optimization in the development of a set of such tools. Efforts in global airlift optimization began at the Naval Postgraduate School in Fiscal Year 1994 with funding from USAF/SAA. These efforts continue to the present time.

In 1994, the work sponsored by USAF/SAA resulted in the THRUPUT2 time-phased global airlift mobility model which was developed during thesis research by Captain Lim Teo-Weng, Singapore Air Force (Lim, 1994). In adding a desired time dimension, Captain Lim's model expanded upon the original THRUPUT - a static model developed by Major Kirk Yost, USAF (Yost, 1994).

### **B. PROBLEM**

THRUPUT2 is a very large linear program due to the huge number of units associated with "real life" scenarios. Recently, for a data set modeling a two MRC scenario, THRUPUT2 took over three hours to generate and solve using GAMS (Brooke et al., 1992) software on an IBM RS6000 computer. Furthermore, this data set was made solvable only by aggregating time periods into steps of two days. The

larger these scenarios are, the more time and memory required to solve the model.

#### C. GOAL

The purpose of this thesis is to make the model smaller and, consequently, reduce the time to solve a given scenario. If the model's size can be sufficiently reduced, time aggregation of the model will be unnecessary.

#### D. METHODOLOGY

The THRUPUT2 formulation (given in Appendix A) contains many variables and constraints that are indexed over units. The model will be smaller if we aggregate into "groups" those units that share origin/destination pairs and have overlapping delivery windows. This treatment requires a reformulation of the current THRUPUT2 model.

The new model will have some limitations. THRUPUT2 had a penalty in the objective function for late deliveries. This feature must be removed from a unit aggregation model because we will not be able to tell which particular unit within a group is having its cargo or troops delivered during an arbitrary time period. Further, the new model will use a weighted cargo density for each group of units. By reducing the densities for those units in a group that have greater densities, this weighting can potentially allow a group to move cargo which would not be movable in THRUPUT2.

To ameliorate the differences in solutions caused by this unit aggregation, we will also develop a second model

which will remove the first model's aggregations. Because only aircraft loading decisions will be necessary for the disaggregation, this secondary model will be a much smaller model than the original THRUPUT2 model, which had to make both loading and aircraft scheduling decisions. This model will incorporate the lateness penalty in its objective function and will use unweighted densities for each unit. While the disaggregation model will require additional time to solve, the time required to generate and solve the Unit Aggregation/Disaggregation Model combination will be less than the time to generate and solve using THRUPUT2.

#### **E. JUSTIFICATION**

Analysts need to run the model many times in order to evaluate such things as different fleet mixes. The time required to solve the model thus becomes a factor in either the quality or timeliness of this analysis. Furthermore, if the model can be made sufficiently small, we can dispose of the time aggregation mentioned previously.



## **II. UNIT AGGREGATION MODEL**

### **A. OVERVIEW**

The Unit Aggregation Model is largely based on the Morton, Rosenthal, and Lim (1995) THRUPUT2 (Appendix A). Like THRUPUT2, the aggregation model has constraints for the following categories: demand satisfaction, aircraft balance, aircraft capacity, aircraft utilization, and airfield capacity. These constraints are modified to reflect the aggregation of units into groups. The Unit Aggregation Model also contains additional constraints which limit delivery. These are referred to as maximum delivery constraints. The objective function of the model will maximize delivery of unit equipment (UE) and passengers (PAX).

### **B. GROUP FORMATION**

The Unit Aggregation Model requires the formation of groups. Those units that share the same origin-destination pair and have overlapping delivery windows form a group. Figure 1 shows the delivery windows for six units with the same origin and destination. Note that the delivery windows of Units A, B, E, and F overlap at least one of the other unit's delivery window, while Units C and D only overlap each other's window. Thus, Group 1 consists of member Units A, B, E, and F and Group 2 contains Units C and D.

It would be inappropriate to allow an individual unit's delivery window to expand to the window of its group. The

Unit Aggregation Model helps prevent this from happening with some new constraints called Maximum Delivery constraints. The complete formulation is presented in the next section.

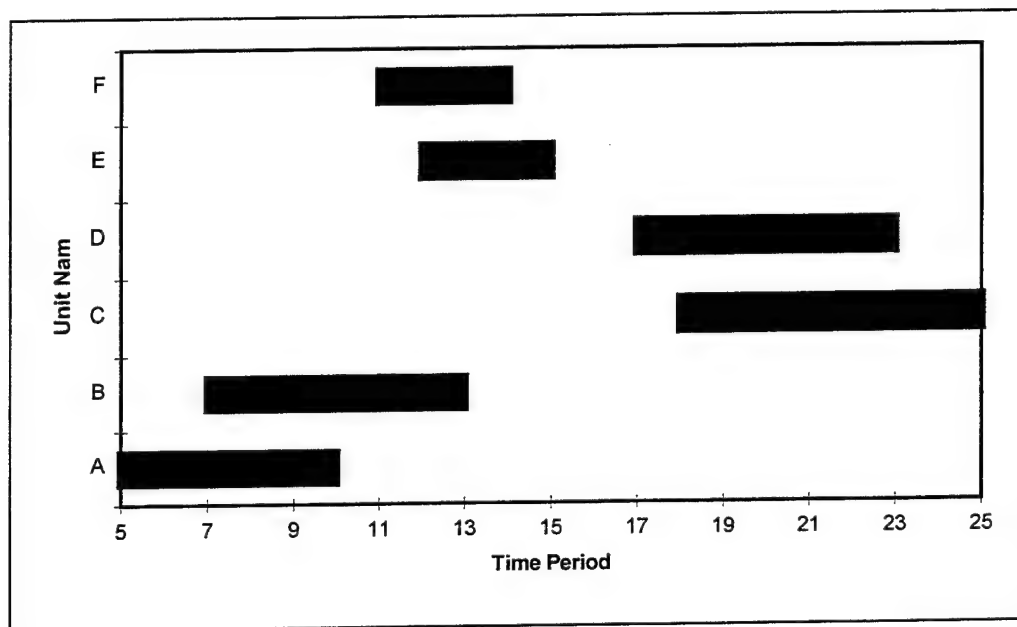


Figure 1. Delivery windows for six different units having the same origin-destination pair.

## C. MATHEMATICAL FORMULATION OF UNIT AGGREGATION MODEL

### 1. Indices

$u$	units, e.g., 82 <sup>nd</sup> Airborne
$g$	groups of units
$a$	aircraft types, e.g., C5, C17
$t, t'$	time periods
$b$	airfields (a/f) in general
$r$	routes

## 2. Index Sets

### a. Airfield Index Sets

$B$  set of all airfields  
 $I \subseteq B$  origin airfields  
 $K \subseteq B$  destination airfields

### b. Aircraft Index Sets

$A$  set of all aircraft (a/c) types  
 $A_{\text{bulk}} \subseteq A$  a/c capable of carrying only bulk-sized cargo  
 $A_{\text{over}} \subseteq A$  a/c capable of carrying bulk- and over-sized cargo, but not out-sized  
 $A_{\text{out}} \subseteq A$  a/c capable of carrying bulk-, over-, and out-sized cargo

### c. Route Index Sets

$R$  set of all routes  
 $R_a \subseteq R$  permissable routes for a/c type  $a$   
 $R_{ab} \subseteq R_a$  permissable routes for a/c type  $a$  that use a/f  $b$   
 $R_{au} \subseteq R_a$  permissable routes for a/c type  $a$  which have an appropriate origin  $i$  and destination  $k$  for unit  $u$   
 $DR_i \subseteq R$  delivery routes that originate from origin  $i$

$RR_k \subseteq R$  recovery routes that originate from  
destination  $k$

**d. Time Index Sets**

$T$  set of all time periods

$T_{gar}$  possible launch times of missions for group  $g$   
using a/c type  $a$  and route  $r$

$AT_u$  all days in delivery window for group to  
which unit  $u$  belongs prior to the unit's ALD

$DT_{uar}$  all days in delivery window for the group to  
which unit  $u$  belongs prior to the ALD of the  
next unit available for loading

$RT_{uar}$  all days in delivery window for group to  
which unit  $u$  belongs in which UE/PAX may be  
sent along route  $r$  on a/c type  $a$  and arrive  
at the destination by unit  $u$ 's RDD

**e. Unit Index Sets**

$U$  Set of all units

$U_g \subseteq U$  All units which belong to group  $g$

$RPR_{gu} \subseteq U_g$  All units belonging to group  $g$  which have an  
RDD prior to or the same as unit  $u$ 's RDD

$APA_{gu}$  All units belonging to group  $g$  which have an  
ALD prior to unit  $u$ 's ALD

First<sub>g</sub>      The unit belonging to group *g* having the  
earliest ALD

### 3.    Data

#### *a.    Movement Requirement Data*

MovePAX<sub>u</sub>      PAX (in 100s) to be moved for unit *u*

MoveUE<sub>u</sub>      UE (in 100 stons) to be moved for unit *u*

ProBulk<sub>u</sub>      proportion of unit *u* UE that is bulk-  
sized

ProOver<sub>u</sub>      proportion of unit *u* UE that is over-  
sized

ProOut<sub>u</sub>      proportion of unit *u* UE that is out-  
sized

#### *b.    Penalty Data*

NoGoPenUE<sub>u</sub>    Non-delivery penalty (per ston) for unit  
*u* equipment

NoGoPenPAX<sub>u</sub>    Non-delivery penalty (per troop) for  
unit *u* PAX

Preserve<sub>at</sub>      Penalty (small artificial cost) for  
keeping a/c type *a* in mobility system at  
time *t*

#### *c.    Cargo Data*

UESqFt<sub>u</sub>      Average floor space requirement per ston of  
UE from unit *u*

$PAXWt_u$  Average weight (in stons) of one troop from unit  $u$ , including personal equipment

**d. Aircraft Data**

$Supply_{at}$  Number of a/c type  $a$  that become available at time  $t$

$MaxPAX_a$  Maximum allowed PAX on a/c type  $a$

$PAXSqFt_a$  Average square footage (in sq. ft.) used by a unit  $u$  troop for a/c type  $a$

$ACSqFt_a$  Available floor space (in sq. ft.) for a/c type  $a$

$LoadEff_a$  Cargo space loading efficiency ( $\leq 1$ ) for a/c type  $a$

$URate_a$  Established utilization rate (flying hours per a/c per day) for a/c type  $a$

**e. Airfield Data**

$MOGCap_{bt}$  A/C handling capacity (in narrow-body equivalents) at airfield  $b$  in time  $t$

$MOGReq_{ab}$  A/C handling capacity consumed by one a/c type  $a$  at airfield  $b$

$MOGEff_{bt}$  MOG efficiency factor ( $\leq 1$ )

**f. Aircraft Route Data**

$MaxLoad_{ar}$  Maximum payload (in stons) for a/c type  $a$  along route  $r$

$FltTime_{ar}$  Flying hours consumed by a/c type  $a$  along route  $r$   
 $GTime_{abr}$  Ground time for a/c type  $a$  at a/f  $b$  flying route  $r$   
 $DTime_{abr}$  Cumulative time (flying time and ground time) taken by a/c type  $a$  to reach a/f  $b$  on route  $r$   
 $CTime_{ar}$  Cumulative time (flying and ground time) taken by a/c  $a$  to fly route  $r$

#### 4. Variables

$X_{gart}$  Number of a/c type  $a$  flying route  $r$  with start time  $t$  airlifting group  $g$   
 $Y_{art}$  Number of a/c type  $a$  recovering along route  $r$  with start time  $t$   
 $ALLOT_{ait}$  Number of a/c type  $a$  allocated to origin  $i$  in time  $t$   
 $RELEASE_{ait}$  Number of a/c type  $a$  allocated to origin  $i$  in time  $t-1$  but not scheduled for any missions from time  $t$  on  
 $H_{ait}$  Number of a/c type  $a$  inventoried at origin  $i$  during time  $t$   
 $HP_{akt}$  Number of a/c type  $a$  inventoried at destination  $k$  during time  $t$

$NPLANES_{at}$  Number of a/c type  $a$  in the airlift system  
 during time  $t$

$TONSUE_{gart}$  Stons of group  $g$  UE sent on a/c type  $a$  along  
 route  $r$  with start time  $t$

$TPAX_{gart}$  Group  $g$  PAX sent on a/c type  $a$  along route  $r$   
 with start time  $t$

$BLKNOGO_u$  Stons of bulk-sized UE from unit  $u$  not sent  
 within the delivery window

$OVRRNOGO_u$  Stons of over-sized UE from unit  $u$  not sent  
 within the delivery window

$OUTNOGO_u$  Stons of out-sized UE from unit  $u$  not sent  
 within the delivery window

$BLKONOVRR_u$  Stons of unit  $u$  bulk-sized cargo sent on  
 aircraft capable of carrying up to over-sized  
 cargo

$BLKONOUT_u$  Stons of unit  $u$  bulk-sized cargo sent on  
 aircraft capable of carrying up to out-sized  
 cargo

$OVRRONOUT_u$  Stons of unit  $u$  over-sized cargo sent on  
 aircraft capable of carrying up to out-sized  
 cargo

$PAXNOGO_u$  Unit  $u$  PAX not sent within the delivery  
 window

## 5. Objective Function

The primary component in the objective function of the model is to minimize the sum of the penalties for undelivered PAX and UE. These penalties are the product of a unit-specific weight and the amount of cargo or troops not sent. The secondary component of the objective function, as discussed in Morton, Rosenthal, and Lim (1995), has much less weight than the primary component; it rewards release of those aircraft no longer required for deliveries. The objective function is:

**Minimize**

$$\sum_u (NoGoPenPax_u * PAXNOGO_u + NoGoPenUE_u * UENOGO_u) + \\ \sum_a \sum_t Preserve_{at} * NPLANES_{at}$$

Because aggregation has eliminated the ability to distinguish deliveries among units within groups, this objective function no longer has a penalty for late deliveries. The length of the delivery window has not been reduced, however. In order to ensure as much UE/PAX are delivered on time as possible, the disaggregation model's objective function will include a lateness penalty, while allowing delivery in the same window as the aggregation model.

## 6. Constraints

### a. Demand Satisfaction

The Unit Aggregation Model has four sets of demand satisfaction constraints. The first three ensure delivery of cargo over each of the cargo classes. These constraints ensure cargo compatibility and account for cargo shipped by aircraft capable of carrying larger sized cargo.

$$\begin{aligned} \sum_{a \in \text{bulk}} \sum_{r \in R_{gu}} \sum_{t \in RT_{uar}} \text{TONSUE}_{gart} + \sum_{u' \in RPR_{gu}} (\text{BLKNOGO}_{u'} + \text{BLKONOV}_{u'} + \text{BLKONOUT}_{u'}) \\ \geq \sum_{u' \in RPR_{gu}} \text{ProBulk}_{u'} * \text{MoveUE}_{u'} \quad , \forall g, u \in U_g \end{aligned}$$

$$\begin{aligned} \sum_{a \in \text{ovr}} \sum_{r \in R_{gu}} \sum_{t \in RT_{uar}} \text{TONSUE}_{gart} + \sum_{u' \in RPR_{gu}} (\text{OVRNOGO}_{u'} + \text{OVRONOUT}_{u'} - \text{BLKONOV}_{u'}) \\ \geq \sum_{u' \in RPR_{gu}} \text{ProOvr}_{u'} * \text{MoveUE}_{u'} \quad , \forall g, u \in U_g \end{aligned}$$

$$\begin{aligned} \sum_{a \in \text{out}} \sum_{r \in R_{gu}} \sum_{t \in RT_{uar}} \text{TONSUE}_{gart} + \sum_{u' \in RPR_{gu}} (\text{OUTNOGO}_{u'} - \text{BLKONOUT}_{u'} - \text{OVRONOUT}_{u'}) \\ \geq \sum_{u' \in RPR_{gu}} \text{ProOut}_{u'} * \text{MoveUE}_{u'} \quad , \forall g, u \in U_g \end{aligned}$$

These three constraints differ from their THRUPUT2 counterparts in that they account for the type of aircraft on which differing classes of cargo are shipped. They also differentiate cargo not sent by its class. These modifications were necessary to allow construction of the maximum delivery constraints, and, incidentally, add some resolution desired by AFSA.

The fourth demand constraint ensures the delivery of passengers.

$$\sum_a \sum_{r \in R_{gu}} \sum_{t \in T_{uar}} TPAX_{gart} + \sum_{u' \in RPR_{gu}} PAXNOGO_{u'} \geq \sum_{u' \in RPR_{gu}} MovePax_{u'}, \forall g, u \in U_g$$

This constraint is basically the same as the corresponding constraint for THRUPUT2.

### **b. Maximum Delivery**

These constraints allow delivery of only UE and PAX which are available for loading at the time period in question. In so doing, they prevent delivery after a unit's RDD (including the extension) or before the unit's ALD. There will be one constraint for each unique ALD in a group, less one for the first ALD. Thus, there will be no maximum delivery constraints written for those groups which have only one unit.

$$\begin{aligned} \sum_{a \in bulk} \sum_{r \in R_{gu}} \sum_{t \in AT_u} TONSUE_{gart} + \sum_{u' \in APA_{gu}} (BLKNOGO_{u'} + BLKONOV_{R_{u'}} + BLKONOUT_{u'}) \\ \leq \sum_{u' \in APA_{gu}} PROBL_{u'} * MoveUE_{u'}, \quad \forall g, u \in U_g, u \neq First_g \end{aligned}$$

$$\begin{aligned} \sum_{a \in ovr} \sum_{r \in R_{gu}} \sum_{t \in AT_u} TONSUE_{gart} + \sum_{u' \in APA_{gu}} (OVRNOGO_{u'} + OVRONOUT_{u'} - BLKONOV_{R_{u'}}) \\ \leq \sum_{u' \in APA_{gu}} ProOvr_{u'} * MoveUE_{u'}, \quad \forall g, u \in U_g, u \neq First_g \end{aligned}$$

$$\begin{aligned} \sum_{a \in out} \sum_{r \in R_{gu}} \sum_{t \in AT_u} TONSUE_{gart} + \sum_{u' \in APA_{gu}} (OUTNOGO_{u'} - BLKONOUT_{u'} - OVRONOUT_{u'}) \\ \leq \sum_{u' \in APA_{gu}} ProOut_{u'} * MoveUE_{u'}, \quad \forall g, u \in U_g, u \neq First_g \end{aligned}$$

$$\sum_a \sum_{r \in R_{gu}} \sum_{t \in AT_u} TPAX_{gart} + \sum_{u' \in APA_{gu}} PAXNOGO_{u'} \leq \sum_{u' \in APA_{gu}} MovePax_{u'}, \quad \forall g, u \in U_g, u \neq First_g$$

The nondelivery variables (e.g., BLKNOGO<sub>u</sub>) and the variables allowing shipment of cargo on aircraft capable of handling larger cargo (e.g., OVRONOUT<sub>u</sub>) are necessary to account for cargo by class and delivery aircraft type. If undelivered cargo is not segregated by class, infeasibilities will occur when the total amount of undelivered cargo is greater than the amount of, say, oversized cargo to be delivered. Further, because it is necessary to account for all deliveries of a certain cargo class in one constraint, variables which permit out of class cargo-to-aircraft matchings are necessary.

### c. A/C Balance

The following five sets of constraints ensure that all aircraft are accounted for and that no more aircraft are used than are in the mobility system at any given time. The first two balance the flow of aircraft at origin and destination airfields, respectively.

$$\sum_g \sum_{r \in DR_i} X_{gar} + H_{ait} + RELEASE_{ait} = H_{ai,t-1} + ALLOT_{ait} + \sum_{r \in R_{ai,t} + [CTime_{ar}] = t} \sum Y_{ar}, \forall a, i, t$$

$$\sum_{r \in RR_k} Y_{ar} + HP_{akt} = HP_{ak,t-1} + \sum_g \sum_{r \in R_{ak}} \sum_{\substack{r' \in I_{gar} \\ t' + [CTime_{ar}] = t}} X_{gar}, \forall a, k, t$$

The third constraint limits the quantity of aircraft allocated to origin airfields to the supply of aircraft.

$$\sum_i ALLOT_{ait} \leq Supply_{ai}, \forall a, t$$

The fourth ensures that aircraft released from the system are no longer used by the model.

$$NPLANES_{at} = \sum_{t'=1}^t \sum_i ALLOT_{ait'} - \sum_{t'=1}^t \sum_i RELEASE_{ait'}, \forall a, t$$

The final aircraft conservation constraint set compensates for problems which arise from rounding  $CTime_{ar}$ .

For a more in-depth discussion of the workings of this constraint, see Morton, Rosenthal, and Lim (1995).

$$\sum_{r \in R_a} \sum_{t'=1}^t K_{art'} * (X_{art'} + Y_{art'}) + \sum_i \sum_{t'=1}^t H_{ait'} + \sum_k \sum_{t'=1}^t HP_{akt'} \leq \sum_{t'=1}^t NPLANES_{at'}, \forall a, t$$

where

$$K_{art'} = \begin{cases} t - t' + 1 & \text{if } t' \leq t < t' + CTime_{ar} - 1 \\ CTime_{ar} & \text{if } t > t' + CTime_{ar} - 1 \end{cases}$$

Except for a change in indices from units to groups, all aircraft balance constraints are identical to their THRUPUT2 counterparts.

#### **d. Troop Carriage**

This constraint set limits the amount of passengers being carried to the number of seats available on the aircraft.

$$TPAX_{gart} \leq MaxPax_a * X_{gart}, \forall g, a, r, t: t \in T_{gar}$$

#### **e. Maximum Payload**

Here, we limit the total payload of aircraft to the weight allowable along the given route.

$$TONSUE_{gart} + PaxWt * TPAX_{gart} \leq MaxLoad_{ar} * X_{gart}, \forall g, a, r, t: t \in T_{gar}$$

#### f. Floor Space

We also limit the amount of floor space for PAX and UE to the total available footage for the aircraft.

$$PaxSqFt_a * TPAX_{garl} + WtdAvgUESqFt_{gt} * TONSUE_{garl} \leq LoadEff_a * X_{garl}, \forall g, a, r, t$$

#### g. A/C Utilization

The total flying hours consumed by the aircraft are limited to a quantity established by the Air Mobility Command.

$$\sum_g \sum_{r \in R_a} \sum_{t \in T_{gar}} FltTime_{ar} * X_{garl} + \sum_{r \in R_a} \sum_t FltTime_{ar} * Y_{arl} \leq \sum_t URate_a * NPLANES_{ar}, \forall a$$

#### h. A/C Handling

This final set of constraints limits the number of flights in and out of an airfield to the handling capacity of the airfield.

$$\begin{aligned} & \sum_g \sum_a \sum_{r \in R_a} \sum_{\substack{t' \in T_{gar} \\ t' + [Dtime_{abr}] = t}} (MOGReq_{ab} * GTime_{abr}) * X_{garl}, \\ & + \sum_a \sum_{r \in R_a} \sum_{\substack{t' \in T_{gar} \\ t' + [Dtime_{abr}] = t}} (MOGReq_{ab} * GTime_{abr}) * Y_{arl} \leq MOGEff_{bl} * MOGCap_{bl}, \forall b, t \end{aligned}$$

### D. IMPACT OF REFORMULATION

Although the Unit Aggregation Model is based on the THRUPUT2 model, several major differences exist between the two. We have aggregated cargo densities for units within groups. Additionally, we have removed some constraints and variables while adding others.

Flight variables ( $X$ ), and troop and cargo delivery variables (TONSUE and TPAX) are now indexed by groups instead of units. This results in a variable reduction for each instance in which more than one unit was being delivered over the same  $(i,k)$  combination during the same time period. However, we have added six variables for each unit while removing one. This addition to the total number of variables is small compared to the reduction allowed when  $X$ , TONSUE, and TPAX are indexed by groups instead of units.

We have added four maximum delivery constraints for each unique ALD for units within a group, less one for the first unit to begin delivery in the group. However, this increase is offset by a significant reduction in the number of aircraft capacity constraints, which are now indexed over groups rather than units. This savings is large because aircraft capacity constraints are also indexed over aircraft types, routes, and time periods.

The aggregation of units into groups has reduced the size of one index set. While additional constraints and variables were required for the reformulation, the reduction in the size of the model is quite significant. For a discussion of this reduction, see the model size comparison in Chapter IV. However, the solution to this model is difficult to interpret when unit resolution is desired. Therefore, we develop a secondary model which disaggregates groups into units.



### **III. DISAGGREGATION MODEL**

In the Unit Aggregation Model, we sacrificed some unit resolution and, thereby, the ability to discern lateness. This chapter develops a small secondary optimization model that takes the Unit Aggregation Model's solution as input and attempts to remedy this shortcoming. This model solves quickly relative to the Unit Aggregation Model. Optimization is necessary here because a new loading assignment of units onto previously scheduled aircraft must be determined. Because we require another optimization model and once again have unit-level resolution, we have reinstated the late delivery penalty in the objective function.

#### **A. WORD FORMULATION**

This model assigns UE and PAX from each unit to flights scheduled by the Unit Aggregation Model subject to the handling capacities of the aircraft. We require four sets of demand satisfaction constraints similar to those in the previous model. We also have three sets of aircraft capacity constraints.

#### **B. MATHEMATICAL FORMULATION**

##### **1. Additional Model Information**

All indices, sets, and data remain the same with the following exceptions:

**a. Index Sets**

$T'_{uar}$  The set of all days in which UE and PAX from unit  $u$  can be sent on a/c type  $a$ , over route  $r$ , and arrive by the unit's required delivery date

**b. Data**

$\bar{x}_{gart}$  The optimal value of the  $x_{gart}$  variable in the unit aggregation model

$DaysLate_{uart}$  The number of days late UE and/or PAX from unit  $u$  would be if it left on a/c type  $a$  in time  $t$  and flew along route  $r$

$LatePenUE_u$  The penalty for each day of lateness for UE from unit  $u$

$LatePenPAX_u$  The penalty for each day of lateness for PAX from unit  $u$

$MaxLate_u$  The maximum allowed number of days late a unit  $u$  delivery is allowed

**c. Variables**

For the disaggregation model, we will use the same delivery and non-delivery variables as in the Unit Aggregation Model with one exception. The TONSUE and TPAX are now indexed over units instead of groups. We use primes to designate this difference.

## 2. Objective Function

We will use an objective function which minimizes non-delivery and late delivery penalties.

**Minimize**

$$\begin{aligned} & \sum_u \sum_a \sum_{r \in R_a} \sum_{t \in T'_{uar}} \text{LatePenUE}_u * \text{DaysLate}_{uar} * \text{TONSUE}'_{uar} + \\ & \sum_u \sum_a \sum_{r \in R_a} \sum_{t \in T'_{uar}} \text{LatePenPAX}_u * \text{DaysLate}_{uar} * \text{TPAX}'_{uar} + \\ & \sum_u (\text{NoGoPenUE}_u * \text{UENOGO}'_u + \text{NoGoPenPAX}_u * \text{PAXNOGO}'_u) \end{aligned}$$

## 3. Constraints

### a. Demand Satisfaction

The four following sets of constraints ensure demand satisfaction for UE and PAX from unit  $u$ . There are four sets; one for each type of cargo and troops.

$$\sum_{a \in A_{bulk}} \sum_{r \in R_{au}} \sum_{t \in T_{uar}} \text{TONSUE}'_{uar} + \text{UENOGO}'_u = \text{MoveUE}_u \quad \forall u: \text{MoveUE}_u > 0$$

$$\sum_{a \in A_{out}} \sum_{r \in R_{au}} \sum_{t \in T_{uar}} \text{TONSUE}'_{uar} + \text{UENOGO}'_u \geq \text{ProOut}_u * \text{MoveUE}_u \quad \forall u: \text{MoveUE}_u > 0$$

$$\sum_{a \in A_{over}} \sum_{r \in R_{au}} \sum_{t \in T_{uar}} \text{TONSUE}'_{uar} + \text{UENOGO}'_u \geq (\text{ProOut}_u + \text{ProOver}_u) * \text{MoveUE}_u$$

$$\forall u: \text{MoveUE}_u > 0$$

$$\sum_a \sum_{r \in R_{au}} \sum_{t \in T_{uar}} \text{TPAX}'_{uar} + \text{PAXNOGO}'_u = \text{MovePAX}_u \quad \forall u: \text{MovePAX}_u > 0$$

### b. Aircraft Capacity

The following constraints ensure that the capacity of the aircraft is not exceeded by the assigned payload.

$$\sum_{u \in U_g} TPAX'_{uarl} \leq MaxPAX_a * \bar{X}_{garl} \quad \forall g, a, r, t: t \in T_{gar}$$

$$\sum_{u \in U_g} TONSUE'_{uarl} + PAXWt * TPAX'_{uarl} \leq MaxLoad_{ar} * \bar{X}_{garl} \quad \forall g, a, r, t: t \in T_{gar}$$

$$\sum_{u \in U_g} (PAXSqFt_a * TPAX'_{uarl} + UESqFt_u * TONSUE'_{uarl}) \leq ACSqFt_a * LoadEff_a * \bar{X}_{garl}$$

$$\forall g, a, r, t: t \in T_{gar}$$

Because the formulation for the disaggregation model merely limits cargo and troops sent to the capacities of the aircraft scheduled by the aggregation model, it will always provide a feasible solution. However, the combination of the two models may yield a suboptimal result. This possibility lies in the use of the weighted densities. Because the weighted density is the same for any unit delivering in a given time period, those units with much higher densities will effectively have their density reduced. This reduction in density may allow the model to deliver more cargo for that unit than it should. The disaggregation model will prevent this excess delivery from taking place. However, this may now give us an unused airlift capacity for the time period. This excess capacity could potentially have been used somewhere else in the model, thus producing a suboptimal solution.

The disaggregation model will solve very quickly because it merely assigns UE and PAX to aircraft missions scheduled by the Unit Aggregation Model. It therefore has no concerns other than demand satisfaction and aircraft

capacity. For additional speed, if needed, the model could be solved as a series of smaller models, one for each group.

While some degree of optimality is sacrificed when using the aggregation/disaggregation model combination, the reduction in model size could become a critical factor in whether or not a model can be solved. The disaggregation model provides the same unit resolution as THRUPUT2, including optimizing delivery for timeliness.



#### IV. PERFORMANCE AND COMPARISON

##### A. PERFORMANCE OF UNIT AGGREGATION/DISAGGREGATION MODEL COMBINATION

For comparison to THRUPUT2, we used the same notional data set used by Lim (described in his thesis). Using GAMS/OSL and an RS6000/590 computer, the unit aggregation/disaggregation model combination took just over 100 seconds to generate and solve the scenario. Solving the same scenario with the same computer and software, THRUPUT2 takes about 43% more time.

##### B. COMPARISON WITH THRUPUT2

Here we compare model size, time requirements, and delivery performance. There are two different data sets used in this comparison. The first is a small, notional data set; the other is larger and based on actual data for a two MRC scenario.

###### 1. Model Size

In Table 1, we see the size differences between the Unit Aggregation Model and THRUPUT2. Table I compares the two models for both the notional data set and the two MRC data set. When solving large models, the largest

	Notional Data Set		Two MRC Data Set	
	Unit Aggregation	THRUPUT2	Unit Aggregation	THRUPUT2
rows	5,793	7,485	26,790	198,237
columns	9,389	11,093	33,175	210,727
nonzeroes	43,017	55,174	189,231	1,217,745

Table 1. A comparison of model size and nonzero quantities for the two models using both data sets.

consumption of memory is due to the number of nonzeros in

the model. Examining the figures for the notional data set, the quantity of nonzeros in the unit aggregation model is about 80% of those in THRUPUT2; for the two MRC Data Set, the quantity of nonzeros is about 15%. The difference in memory savings for the two data sets is actually quite dramatic (20% for the notional data set versus over 80% for the more realistic scenario). The explanation for this difference is in the nature of the data sets. The notional data set has nine different  $(i,k)$  pairs for twenty units. The more realistic data set has over 220 units and 23 different origin/desination pairs.

## **2. Time Requirements**

The time savings for each data set is significant. For the notional data set, THRUPUT2 takes about 40 seconds to generate and 100 seconds to solve on the RS6000. The unit aggregation model takes 35 seconds to generate and 65 seconds to solve, and the disaggregation model requires 15 seconds for generation and 20 seconds for solving. For the two MRC scenario, the difference in time to generate and solve is more dramatic. It takes an hour to generate and over two hours to solve using THRUPUT2. The unit aggregation model took six minutes to generate and ten minutes to solve, and the disaggregation model required only one minute for generation and three minutes to arrive at a solution. The time required for generating and solving the scenario is reduced by almost 90%.

### 3. Delivery Results

Figure 2 compares the cumulative delivery results for each model with the cumulative demand for the notional scenario. The cargo delivery profile for each model is quite similar. Large differences in the cargo densities for units in the same groups cause the discrepancy between the two delivery profiles. In the unit aggregation model, these densities were weighted for each group. This weighting caused the group's density to be less than some member units' cargo densities. The difference made it impossible to ship as much UE during a unit's delivery window given the flights scheduled by the unit aggregation model.

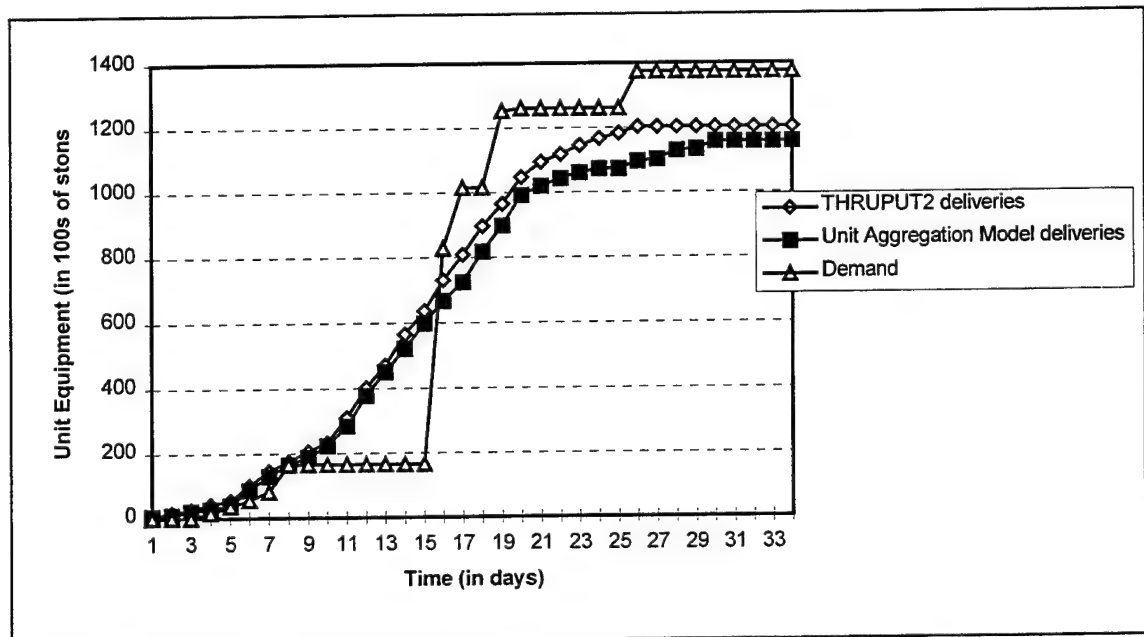


Figure 2. A display of cumulative deliveries of unit equipment for the two models compared to the cumulative demand for the notional scenario. Note the differences in the delivery profiles. This is due to the use of weighted densities for units within groups.

Figure 3 shows the same delivery profiles for the two MRC scenario. Note that the aggregation/disaggregation

model combination allowed more deliveries than THRUPUT2. This delivery profile created by the Unit Aggregation Model is not a feasible solution for THRUPUT2. This difference requires more extensive analysis, but is likely due to the use of cumulative demand satisfaction and maximum delivery constraints.

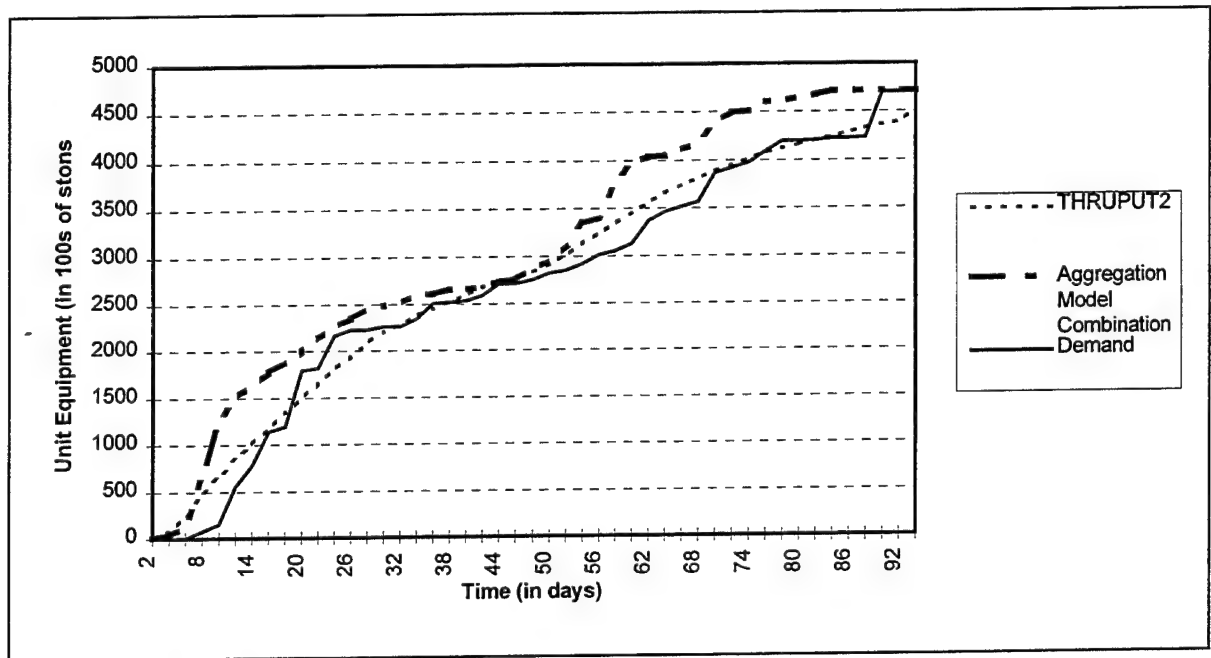


Figure 3. A display of cumulative deliveries of unit equipment for the two models compared to the cumulative demand for the two MRC scenario.

Unit closure is a metric U.S. Air Force uses to make comparisons between model runs. A unit is considered closed when all unit equipment for that unit has been shipped to its destination. The difference in closure for the notional data set is significant. While THRUPUT2 fails to close only two units, the aggregation/disaggregation model combination fails to close ten. However, this metric may be somewhat misleading. Neither model is actually optimizing closure.

It may be possible to close more units in either scenario by shifting cargo loading from one unit to another.



## **V. CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS**

### **A. CONCLUSIONS**

The Unit Aggregation/Disaggregation Model combination can be used to solve the same scenarios as THRUPUT2. The combination is smaller and takes less time to generate and solve. It also may alleviate the need to aggregate time in order to solve a scenario. In these respects, the combination forms a better model.

In addition to the memory savings, the model also now requires less time to generate and solve. This brings the model back in line with its original intentions of being a "quick turn" model.

### **B. LIMITATIONS**

When we put units and their associated UE and PAX through the aggregation/disaggregation, we lose the resolution we had in THRUPUT2. For example, as cited earlier in Chapter IV, we may cause problems by aggregating units with significantly different cargo densities. Further, the model combination may allow more deliveries than is feasible for THRUPUT2.

### **C. RECOMMENDATIONS**

The Unit Aggregation Model can be recommended because of its tremendous savings in time and reduction in model size. With the time savings, we get the flexibility to examine several different scenarios in a much shorter time span. As a result of the size reduction, we might also add

features to the model's capabilities such as aerial refueling or intra-theater cargo delivery. However, use of the Unit Aggregation Model requires some caution. Due to problems noted in previous sections, we should only aggregate units with similar densities. This will increase the time and size to some degree, but will still form a leaner, faster model than THRUPUT2.

## APPENDIX A. MATHEMATICAL FORMULATION OF THRUPUT2

The following mathematical formulation of THRUPUT2 is excerpted from Morton, Rosenthal, and Lim [1995].

### Indices

$u$	indexes units, e.g., 82nd Airborne
$a$	indexes aircraft types, e.g., C5, C141
$t, t'$	index time periods
$b$	indexes all airfields (origins, enroutes and destinations)
$i$	indexes origin airfields
$k$	indexes destination airfields
$r$	indexes routes

### Index Sets

#### Airfield Index Sets

$B$	set of available airfields
$I \subseteq B$	origin airfields
$K \subseteq B$	destination airfields

#### Aircraft Index Sets

$A$	set of available aircraft types
$A_{bulk} \subseteq A$	aircraft capable of hauling bulk-sized cargo
$A_{over} \subseteq A_{bulk}$	aircraft capable of hauling over-sized cargo
$A_{out} \subseteq A_{over}$	aircraft capable of hauling out-sized cargo

Bulk cargo is palletized on 88 x 108 inch platforms and can fit on any military aircraft (as well as the cargo-configured 747). Over-sized cargo is non-palletized rolling stock: it is larger than bulk cargo and can fit on a C141, C5 or C17. Out-sized cargo is very large non-palletized cargo that can fit into a C5 or C17 but not a C141.

#### Route Index Sets

$R$	set of available routes
$R_a \subseteq R$	permissible routes for aircraft type $a$
$R_{ab} \subseteq R_a$	permissible routes for aircraft type $a$ that use airfield $b$
$R_{aik} \subseteq R_a$	permissible routes for aircraft type $a$ that have origin $i$ and destination $k$
$DR_i \subseteq R$	delivery routes that originate from origin $i$
$RR_k \subseteq R$	recovery routes that originate from destination $k$

A delivery route is a route flown from a specific unit's origin to its destination for the purpose of delivering cargo and/or passengers. A recovery route is a route flown from a unit's destination to that unit's

or some other unit's origin, for the purpose of making another delivery. Since recovery flights carry much less weight than deliveries, the recovery routes from  $k$  to  $i$  may have fewer enroute stops than the delivery routes from  $i$  to  $k$ .

#### Time Index Sets

$T$  set of time periods  
 $T_{uar} \subseteq T$  possible launch times of missions for unit  $u$  using aircraft type  $a$  and route  $r$

The set  $T_{uar}$  covers the allowed time window for unit  $u$ , which starts on the unit's available-to-load date and ends on the unit's required delivery date, plus some extra time up to the maximum allowed lateness for the unit.

#### Given Data

##### Movement Requirements Data

$MovePAX_{uik}$  Troop movement requirement for unit  $u$  from origin  $i$  to destination  $k$   
 $MoveUE_{uik}$  Equipment movement requirement in short tons (stons) for unit  $u$  from origin  $i$  to destination  $k$   
 $ProBulk_u$  Proportion of unit  $u$  cargo that is bulk-sized  
 $ProOver_u$  Proportion of unit  $u$  cargo that is over-sized  
 $ProOut_u$  Proportion of unit  $u$  cargo that is out-sized

##### Penalty Data

$LatePenUE_u$  Lateness penalty (per ston per day) for unit  $u$  equipment  
 $LatePenPAX_u$  Lateness penalty (per soldier per day) for unit  $u$  troops  
 $NoGoPenUE_u$  Non-delivery penalty (per ston) for unit  $u$  equipment  
 $NoGoPenPAX_u$  Non-delivery penalty (per soldier) for unit  $u$  troops  
 $MaxLate$  Maximum allowed lateness (in days) for delivery  
 $Preserve_{at}$  Penalty (small artificial cost) for keeping aircraft type  $a$  in mobility system at time  $t$

### Cargo Data

$UESqFt_u$	Average cargo floor space (in sq. ft.) per ston of unit $u$ equipment
$PAXWt_u$	Average weight of a unit $u$ soldier inclusive of personal equipment

### Aircraft Data

$Supply_{at}$	Number of aircraft of type $a$ that become available at time $t$
$MaxPAX_a$	Maximum troop carriage capacity of aircraft type $a$
$PAXSqFt_{ua}$	Average cargo space (in sq. ft.) consumed by a unit $u$ soldier for aircraft type $a$
$ACSqFt_a$	Cargo floor space (in sq. ft.) of aircraft type $a$
$LoadEff_a$	Cargo space loading efficiency ( $<1$ ) for aircraft type $a$ . This accounts for the fact that it is not possible in practice to fully utilize the cargo space.
$URate_a$	Established utilization rate (flying hours per aircraft per day) for aircraft type $a$

### Airfield Data

$MOGCap_{bt}$	Aircraft capacity (in narrow-body equivalents) at airfield $b$ in time $t$
$MOGReq_{ab}$	Conversion factor to narrow-body equivalents for one aircraft of type $a$ at airfield $b$
$MOGEff_{bt}$	MOG efficiency factor ( $<1$ ), to account for the fact that it is impossible to fully utilize available MOG capacity due to randomness of ground times

### Aircraft Route Performance Data

$MaxLoad_{ar}$	Maximum payload (in stons) for aircraft type $a$ flying route $r$ .
$GTime_{abr}$	Aircraft ground time (due to onload or offload of cargo, refueling, maintenance, etc.) needed for aircraft type $a$ at airfield $b$ on route $r$

$DTime_{abr}$	Cumulative time (flight time plus ground time) taken by aircraft type $a$ to reach airfield $b$ along route $r$
$FltTime_{ar}$	Total flying hours consumed by aircraft type $a$ on route $r$
$CTime_{ar}$	Cumulative time (flight time plus ground time) taken by aircraft type $a$ on route $r$
$DaysLate_{uart}$	Number of days late unit $u$ 's requirement would be if delivered by aircraft type $a$ via route $r$ with mission start time $t$

### Decision Variables

#### Mission Variables

$X_{uart}$	Number of aircraft of type $a$ that airlift unit $u$ via route $r$ with mission start time during period $t$
$Y_{art}$	Number of aircraft of type $a$ that recover from a destination airfield via route $r$ with start time during period $t$

#### Aircraft Allocation and De-allocation Variables

$ALLOT_{ait}$	Number of aircraft of type $a$ that are allocated to origin $i$ at time $t$
$RELEASE_{ait}$	Number of aircraft of type $a$ that were allocated to origin $i$ prior to time $t$ but are not scheduled for any missions from time $t$ on

#### Aircraft Inventory Variables

$H_{ait}$	Number of aircraft of type $a$ inventoried at origin $i$ at time $t$
$HP_{akt}$	Number of aircraft of type $a$ inventoried at destination $k$ at time $t$
$NPLANES_{at}$	Number of aircraft of type $a$ in the air mobility system at time $t$

#### Airlift Quantity Variables

$TONSUE_{uart}$	Total stons of unit $u$ equipment airlifted by aircraft of type $a$ via route $r$ with mission start time during period $t$
-----------------	---

$TPAX_{uart}$  Total number of unit  $u$  troops airlifted by aircraft of type  $a$  via route  $r$  with mission start time during period  $t$

#### Elastic (Nondelivery) Variables

$UENOGO_{uik}$  Total stons of unit  $u$  equipment with origin  $i$  and destination  $k$  that is not delivered in the prescribed time frame

$PAXNOGO_{uik}$  Number of unit  $u$  troops with origin  $i$  and destination  $k$  who are not delivered in the prescribed time frame

#### Formulation of the Objective Function

Minimize

$$\begin{aligned} & \sum_u \sum_a \sum_{r \in R_a} \sum_{t \in T_{uar}} LatePenUE_u * DaysLate_{uar} * TONSUE_{uar} \\ & + \sum_u \sum_a \sum_{r \in R_a} \sum_{t \in T_{uar}} LatePenPAX_u * DaysLate_{uar} * TPAX_{uar} \\ & + \sum_u \sum_i \sum_k (NoGoPenUE_u * UENOGO_{uik} + NoGoPenPAX_u * PAXNOGO_{uik}) \\ & + \sum_a \sum_l Preserve_{al} * NPLANES_{al} \end{aligned}$$

The  $DaysLate_{uar}$  penalty parameter has value zero if  $t+CTime_{ar}$  is within the prescribed time window for unit  $u$ . Thus, the first two terms of the objective function take effect only when a delivery is late. The third term in the objective function corresponds to cargo and passengers that cannot be delivered even within the permitted lateness. Late delivery and non-delivery occur only when airlift assets are insufficient for on-time delivery.

#### Formulation of the Constraints

There are five categories of constraints. Their mathematical formulations are as follows.

##### Demand Satisfaction Constraints

These constraints ensure demand satisfaction and account for the desired delivery time-windows by use of the index sets  $T_{uar}$  and the lateness parameters  $DaysLate_{uar}$ .

Demand Satisfaction Constraints for All Classes of Cargo:

$$\sum_{a \in A_{bulk}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TONSUE_{uar} + UENOGO_{uik} = MoveUE_{uik} \quad \forall u, i, k: MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Out-Sized Cargo:

$$\sum_{a \in A_{out}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TONSUE_{uar} + UENOGO_{uik} \geq ProOut_u * MoveUE_{uik}$$

$$\forall u, i, k: MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Over-Sized Cargo:

$$\sum_{a \in A_{ovr}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TONSUE_{uar} + UENOGO_{uik} \geq (ProOver_u + ProOut_u) * MoveUE_{uik}$$

$$\forall u, i, k: MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Troops:

$$\sum_a \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TPAX_{uar} + PAXNOGO_{uik} = MovePAX_{uik} \quad \forall u, i, k: MovePAX_{uik} > 0$$

**Aircraft Balance Constraints**

Aircraft Balance Constraints at Origin Airfields:

$$\begin{aligned} \sum_u \sum_{r \in DR_i} X_{uar} + H_{ait} + RELEASE_{ait} &= H_{ai,t-1} + ALLOT_{ait} \\ &+ \sum_{r \in R_{ai,t'} + [CTime_{ar}] = t} Y_{ar,t'} \quad \forall a, i, t \end{aligned}$$

where  $[CTime_{ar}]$  is  $CTime_{ar}$  rounded to the nearest integer.

Aircraft Balance Constraints at Destination Airfields:

$$\sum_{r \in RR_k} Y_{ar,t} + HP_{akt} = HP_{ak,t-1} + \sum_u \sum_{r \in R_{ak}} \sum_{\substack{t' \in T_{uar} \\ t' + [CTime_{ar}] = t}} X_{uar,t'} \quad \forall a, k, t$$

Aircraft Balance Constraints for Allocations to Origins:

$$\sum_{t'=1}^t \sum_i ALLOT_{ait'} \leq \sum_{t'=1}^t Supply_{at} \quad \forall a, t$$

This constraint is in the cumulative form, rather than in the simpler form  $S_i Allot_{ait} \leq Supply_{at}$ , to allow aircraft that become available in period  $t$  to be put into service at a later period.

Aircraft Balance Constraints Accounting for Allocations and Releases:

$$NPLANES_{at} = \sum_{t'=1}^t \sum_i ALLOT_{ait'} - \sum_{t'=1}^t \sum_i RELEASE_{ait'} \quad \forall a, t$$

Cumulative Aircraft Balance Constraints:

$$\begin{aligned} & \sum_{r \in R_a} \sum_{t'=1}^t \sum_u K_{art'} * X_{uart'} + \sum_{r \in R_a} \sum_{t'=1}^t K_{art'} + \sum_i \sum_{t'=1}^t H_{ait'} \\ & + \sum_k \sum_{t'=1}^t HP_{akt'} \leq \sum_{t'=1}^t NPLANES_{at'} \end{aligned}$$

where

$$K_{art'} = \begin{cases} t - t' + 1 & \text{if } t' \leq t < t' + CTime_{ar} - 1 \\ CTime_{ar} & \text{if } t \geq t' + CTime_{ar} - 1 \end{cases}$$

**Aircraft Capacity Constraints**

Troop Carriage Capacity Constraints:

$$TPAX_{uart} \leq MaxPAX_a * X_{uart} \quad \forall u, a, r, t: t \in T_{uar}$$

Maximum Payload Constraints:

$$TONSUE_{uart} + PAXWt * TPAX_{uart} \leq MaxLoad_{ar} * X_{uart} \quad \forall u, a, r, t: t \in T_{uar}$$

Cargo Floor Space Constraints:

$$PAXSqFt_a * TPAX_{uart} + UESqFt_u * TONSUE_{uart} \leq ACSqFt_a * LoadEff_a * X_{uart}$$

$$\forall u, a, r, t: t \in T_{uar}$$

### Aircraft Utilization Constraints

$$\begin{aligned} & \sum_u \sum_{r \in R_a} \sum_{t \in T_{ugr}} FltTime_{ar} * X_{uar} + \sum_{r \in R_a} \sum_t FltTime_{ar} * Y_{ar} \\ & \leq \sum_t URate_a * NPlanes_{at} \quad \forall a \end{aligned}$$

### Aircraft Handling Capacity of Airfields (MOG Constraint)

$$\begin{aligned} & \sum_u \sum_a \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + [DTime_{abr}] = t}} (MOGReq_{ab} * GTime_{abr} / 24) * X_{uar} \\ & + \sum_a \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + [DTime_{abr}] = t}} (MOGReq_{ab} * GTime_{abr} / 24) * Y_{ar} \\ & \leq MOGEff_{bt} * MOGCap_{bt} \quad \forall b, t \end{aligned}$$

## APPENDIX B. RESULTS FOR NOTIONAL DATA SET

### 1. Results from Unit Aggregation/Disaggregation Model combination.

	CLOSE DATE	ALD	RDD	ON TIME	LATE DELIVER	NO GO	TOTAL
UNITA	20.12	8.00	17.00	81.07	105.70		86.77
UNITB	22.32	16.00	20.00	6.92			6.92
UNITC	NA	3.00	8.00	3.79	36.80	22.64	63.23
UNITD	NA	21.00	26.00	39.20	61.19	14.55	114.94
UNITE	NA	3.00	16.00	140.33	66.44	125.07	331.85
UNITE2	NA	3.00	16.00	236.58	87.85	7.41	331.84
UNITF	NA	11.00	19.00	158.90	72.12	7.24	238.27
UNITG	1.53	1.00	4.00	5.03			5.03
UNITH	NA	1.00	4.00			10.13	10.13
UNITI	3.40	2.00	5.00	5.03			5.03
UNITJ	NA	2.00	5.00			10.13	10.13
UNITK	6.39	2.00	5.00	3.10	1.93		5.03
UNITL	9.39	2.00	6.00	1.68	3.23		4.91
UNITM	8.39	3.00	6.00	3.83	1.39		5.22
UNITN	10.39	3.00	6.00	1.93	3.10		5.03
UNITO	NA	3.00	6.00	4.91			4.91
UNITP	NA	3.00	7.00			5.22	5.22
UNITQ	7.83	4.00	7.00	10.13			10.13
UNITR	7.84	4.00	7.00	10.13			10.13
UNITS	8.84	4.00	8.00	10.13			10.13
UNITT	NA	4.00	8.00			10.13	10.13
TOTAL				710.86	446.68	217.43	1374.98

### 2. Results from THRUPUT2.

	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	NO GO	TOTAL
UNITA	21.12	8.00	17.00	142.55	44.22		186.77
UNITB	21.32	16.00	20.00		6.92		6.92
UNITC	NA	3.00	8.00	18.70	28.07	16.46	63.23
UNITD	26.84	21.00	26.00	114.94			114.94
UNITE	NA	3.00	16.00	155.04	20.51	156.30	331.84
UNITE2	20.53	3.00	16.00	191.84	140.00		331.85
UNITF	22.83	11.00	19.00	188.80	49.47		238.27
UNITG	2.32	1.00	4.00	5.03			5.03
UNITH	2.12	1.00	4.00	10.13			10.13
UNITI	5.40	2.00	5.00	5.03			5.03
UNITJ	7.06	2.00	5.00	8.71	1.42		10.13
UNITK	5.37	2.00	5.00	5.03			5.03
UNITL	6.39	2.00	6.00	4.91			4.91
UNITM	6.37	3.00	6.00	5.22			5.22
UNITN	6.39	3.00	6.00	5.03			5.03
UNITO	6.35	3.00	6.00	4.91			4.91
UNITP	11.39	3.00	7.00	2.60	2.62		5.22

UNITQ	7.84	4.00	7.00	10.13			10.13
UNITR	7.84	4.00	7.00	10.13			10.13
UNITS	8.83	4.00	8.00	10.13			10.13
UNITT	10.06	4.00	8.00	5.24	4.89		10.13
TOTAL				904.10	298.12	172.76	1374.98

## APPENDIX C. RESULTS FOR THE TWO MRC DATA SET

### 1. Results using the aggregation/disaggregation model combination.

	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	TOTAL
0199	2.51		2.00	2.06		2.06
0204	4.57	4.00	4.00	0.12		0.12
0205	4.45	4.00	4.00	0.12		0.12
0206	4.47	4.00	4.00	0.12		0.12
0221	2.42	2.00	8.00	2.18		2.18
0223	2.40	2.00	8.00	0.29		0.29
0237	6.13	4.00	8.00	5.67		5.67
0239	6.51	4.00	8.00	50.19	1.91	52.10
0256	8.54	4.00	8.00	0.35		0.35
0260	8.32	4.00	8.00	0.44		0.44
0262	6.66	4.00	8.00	0.49		0.49
0264	8.13	4.00	8.00	0.18		0.18
0276	8.41	6.00	8.00	0.29		0.29
0278	6.70	4.00	8.00	4.78		4.78
0293	8.51	6.00	8.00	1.17		1.17
0332	8.78	8.00	8.00	0.19		0.19
0341	8.41	4.00	8.00	3.15		3.15
0353	10.71	6.00	10.00	55.25		55.25
0368	22.47	22.00	22.00	11.74		11.74
0450	8.42	6.00	10.00	17.18		17.18
0459	8.78	8.00	10.00	3.44		3.44
0465	6.13	4.00	10.00	4.49		4.49
0467	8.13	8.00	10.00	0.43		0.43
0536	16.55	6.00	12.00	34.73	8.15	42.88
0571	8.13	8.00	12.00	3.04		3.04
0573	12.08	8.00	12.00	4.00		4.00
0575	8.11	8.00	12.00	2.46		2.46
0576	6.45	4.00	12.00	11.38		11.38
0580	10.57	6.00	12.00	7.01		7.01
0581	6.13	6.00	12.00	6.24		6.24
0651	12.64	8.00	12.00	6.17		6.17
0655	8.47	6.00	12.00	25.53		25.53
0658	12.60	8.00	12.00	290.97		290.97
0761	12.40	8.00	14.00	2.74		2.74
0796	10.66	8.00	14.00	145.05		145.05
0799	10.78	10.00	14.00	12.45		12.45
0887	12.47	8.00	14.00	37.19		37.19
0888	8.54	8.00	14.00	9.10		9.10
0907	8.13	8.00	14.00	2.31		2.31
0910	8.08	8.00	14.00	0.40		0.40
0940	8.63	8.00	14.00	14.61		14.61
0944	14.35	8.00	16.00	0.04		0.04
0953	12.47	8.00	16.00	57.03		57.03
0954	8.57	8.00	16.00	11.75		11.75
0984	20.45	8.00	16.00	15.57	6.58	22.15
1030	12.08	8.00	16.00	3.86		3.86

1032	16.13	8.00	16.00	1.92		1.92
1079	16.63	8.00	16.00	263.48		263.48
1085	12.55	8.00	16.00	3.99		3.99
1173	18.13	6.00	18.00	21.34		21.34
1195	14.67	8.00	18.00	1.11		1.11
1206	10.78	8.00	18.00	15.83		15.83
1210	8.51	8.00	18.00	10.24		10.24
1268	14.63	8.00	20.00	0.30		0.30
1271	16.26	8.00	20.00	9.18		9.18
1273	12.60	8.00	20.00	17.74		17.74
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	TOTAL
1290	12.54	8.00	20.00	15.62		15.62
1302	12.35	10.00	20.00	0.50		0.50
1312	12.41	8.00	20.00	46.30		46.30
1318	20.60	10.00	20.00	4.63		4.63
1336	8.60	8.00	20.00	13.17		13.17
1349	16.13	16.00	20.00	3.46		3.46
1495	24.60	8.00	20.00	268.58	174.27	442.85
1497	22.45	10.00	20.00	46.95	2.59	49.54
1508	26.43	14.00	22.00	0.22	0.81	1.03
1519	20.13	4.00	22.00	0.55		0.55
1564	26.45	12.00	22.00	11.99	2.07	14.06
1619	8.60	8.00	24.00	4.80		4.80
1658	12.64	8.00	24.00	19.48		19.48
1670	10.64	8.00	24.00	67.03		67.03
1692	18.50	10.00	24.00	12.16		12.16
1721	14.64	10.00	24.00	0.30		0.30
1756	12.60	10.00	24.00	16.35		16.35
1800	10.57	8.00	24.00	48.59		48.59
1842	8.47	8.00	24.00	5.76		5.76
1907	28.60	12.00	24.00	50.19	110.42	160.61
1908	22.45	12.00	24.00	14.10		14.10
1909	30.53	8.00	26.00		15.34	15.34
1941	4.13	4.00	26.00	0.49		0.49
1943	14.47	8.00	26.00	43.57		43.57
2024	10.55	8.00	30.00	0.57		0.57
2032	20.50	16.00	30.00	2.09		2.09
2069	28.63	20.00	30.00	27.94		27.94
2130	8.11	4.00	34.00	0.21		0.21
2135	32.47	10.00	34.00	2.55		2.55
2154	8.72	8.00	34.00	75.92		75.92
2158	26.41	16.00	34.00	4.74		4.74
2184	34.08	26.00	34.00	0.37		0.37
2190	26.43	20.00	34.00	2.26		2.26
2201	32.42	8.00	36.00	1.74		1.74
2259	38.43	16.00	36.00	126.73	1.86	128.59
2267	40.57	24.00	36.00	24.18	4.42	28.60
2268	34.08	28.00	36.00	0.37		0.37
2339	38.57	14.00	38.00	4.38		4.38
2342	30.63	28.00	38.00	13.83		13.83
2354	12.42	10.00	40.00	0.48		0.48
2355	26.43	24.00	40.00	1.25		1.25
2361	26.42	8.00	40.00	9.64		9.64

2390	10.70	8.00	42.00	34.27		34.27
2409	16.55	8.00	42.00	17.38		17.38
2423	42.50	36.00	42.00	0.27		0.27
2484	48.51	18.00	44.00	108.50	13.98	122.48
2565	10.78	10.00	48.00	7.17		7.17
2625	48.53	44.00	48.00	3.96		3.96
2628	44.24	44.00	48.00	0.81		0.81
2637	16.35	8.00	48.00	0.54		0.54
2658	24.55	14.00	48.00	1.02		1.02
2661	20.55	8.00	48.00	8.20		8.20
2675	48.56	38.00	48.00	2.57		2.57
2690	44.41	42.00	48.00	0.85		0.85
2702	46.49	42.00	48.00	0.95		0.95
2705	48.50	44.00	48.00	0.27		0.27
2729	46.55	46.00	48.00	1.50		1.50
2739	46.24	46.00	48.00	8.98		8.98
2740	48.23	46.00	48.00	3.07		3.07
2742	44.24	44.00	48.00	2.04		2.04
2756	46.56	40.00	50.00	15.17		15.17
2760	50.50	44.00	50.00	3.04		3.04
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	TOTAL
2778	50.57	20.00	50.00	27.04	13.31	40.35
2802	46.55	46.00	50.00	0.96		0.96
2808	48.50	42.00	50.00	12.15		12.15
2813	50.50	44.00	50.00	8.73		8.73
2817	50.24	48.00	50.00	1.04		1.04
2819	46.24	46.00	50.00	1.52		1.52
2848	52.49	48.00	52.00	1.71		1.71
2910	46.63	46.00	52.00	2.72		2.72
2915	48.24	48.00	52.00	4.20		4.20
2918	52.19	48.00	52.00	3.57		3.57
2923	48.63	48.00	52.00	14.52		14.52
2926	50.19	48.00	52.00	3.02		3.02
2965	54.56	52.00	54.00	3.03		3.03
3005	54.50	52.00	54.00	4.04		4.04
3021	54.24	54.00	54.00	0.51		0.51
3031	50.41	10.00	54.00	12.91		12.91
3047	52.49	50.00	54.00	37.54		37.54
3107	54.63	54.00	54.00	0.42		0.42
3119	50.24	50.00	54.00	3.84		3.84
3124	54.63	54.00	54.00	3.43		3.43
3153	56.54	50.00	56.00	27.50		27.50
3192	54.59	54.00	56.00	0.28		0.28
3201	54.55	52.00	56.00	47.88		47.88
3256	56.24	54.00	56.00	0.32		0.32
3260	54.50	48.00	56.00	9.38		9.38
3261	54.55	50.00	56.00	13.11		13.11
3265	58.53	20.00	58.00	26.99		26.99
3280	54.56	54.00	58.00	3.16		3.16
3347	56.50	54.00	58.00	0.28		0.28
3394	58.47	52.00	58.00	2.45		2.45
3399	54.63	50.00	58.00	5.54		5.54
3567	60.19	54.00	60.00	1.27		1.27

3602	58.50	48.00	60.00	22.82		22.82
3603	60.50	54.00	60.00	4.17		4.17
3663	60.19	54.00	60.00	14.78		14.78
3664	58.19	54.00	60.00	6.28		6.28
3667	56.52	54.00	60.00	2.19		2.19
3670	60.19	54.00	60.00	16.09		16.09
3690	60.50	54.00	60.00	9.11		9.11
3701	58.53	54.00	62.00	3.30		3.30
3766	62.24	54.00	62.00	3.08		3.08
3789	62.53	52.00	62.00	6.01		6.01
3828	58.53	52.00	62.00	41.21		41.21
3832	58.24	54.00	62.00	30.14		30.14
3840	56.56	52.00	62.00	12.40		12.40
3849	60.49	52.00	62.00	139.77		139.77
3911	60.50	52.00	64.00	73.66		73.66
3925	58.50	14.00	64.00	18.68		18.68
3927	54.63	54.00	64.00	4.08		4.08
3995	66.50	50.00	66.00	37.14		37.14
4006	66.49	54.00	66.00	15.22		15.22
4027	54.24	54.00	66.00	0.53		0.53
4158	68.45	52.00	68.00	33.83		33.83
4165	68.56	58.00	68.00	0.69		0.69
4166	60.57	56.00	68.00	0.85		0.85
4168	60.51	54.00	68.00	5.79		5.79
4169	66.55	54.00	68.00	4.85		4.85
4225	68.51	52.00	70.00	66.76		66.76
4231	68.24	54.00	70.00	2.45		2.45
4238	58.55	52.00	70.00	232.21		232.21
4254	62.54	54.00	70.00	2.74		2.74
4267	66.54	8.00	72.00	0.57		0.57
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	TOTAL
4388	68.57	64.00	74.00	8.34		8.34
4418	64.57	52.00	74.00	25.48		25.48
4423	74.24	54.00	74.00	18.89		18.89
4430	68.50	56.00	74.00	4.81		4.81
4455	76.50	54.00	76.00	95.26		95.26
4470	74.73	56.00	76.00	8.45		8.45
4494	68.63	52.00	76.00	9.27		9.27
4497	68.47	54.00	76.00	4.01		4.01
4511	72.54	54.00	78.00	94.41		94.41
4518	76.56	50.00	78.00	8.04		8.04
4522	66.54	8.00	78.00	0.57		0.57
4558	72.47	54.00	80.00	4.23		4.23
4564	72.50	54.00	80.00	0.53		0.53
4579	82.56	54.00	82.00	2.53		2.53
4724	76.55	50.00	84.00	3.09		3.09
4726	80.56	54.00	84.00	19.01		19.01
4732	80.55	58.00	84.00	3.57		3.57
4814	84.55	54.00	88.00	5.04		5.04
4887	84.50	54.00	90.00	0.73		0.73
5027	70.49	54.00	90.00	0.46		0.46
5045	82.45	54.00	90.00	88.76		88.76
5186	76.47	54.00	90.00	4.47		4.47

5206	82.50	54.00	90.00	26.84		26.84
5243	72.49	50.00	90.00	110.52		110.52
5286	84.57	54.00	90.00	205.40		205.40
5289	66.54	8.00	90.00	5.70		5.70
5299	84.49	44.00	90.00	25.52		25.52
5304	66.50	52.00	90.00	1.98		1.98
4270	62.57	52.00	72.00	24.58		24.58
4277	72.49	54.00	72.00	26.34		26.34
4366	60.59	60.00	74.00	0.05		0.05
TOTAL				4361.01	355.70	4716.71

## 2. Results using THRUPUT2.

	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	NO GO	TOTAL
0199	4.63		4.00	2.06			2.06
0204	4.58	4.00	4.00	0.12			0.12
0205	4.45	4.00	4.00	0.12			0.12
0206	4.47	4.00	4.00	0.12			0.12
0221	8.56	2.00	8.00	2.17			2.17
0223	6.41	2.00	8.00	0.29			0.29
0237	8.13	4.00	8.00	5.67			5.67
0239	6.48	4.00	8.00	50.19			50.19
0256	10.55	4.00	10.00	0.35			0.35
0260	6.41	4.00	10.00	0.44			0.44
0262	10.65	4.00	10.00	0.49			0.49
0264	10.13	4.00	10.00	0.18			0.18
0276	6.47	6.00	10.00	0.29			0.29
0278	6.63	4.00	10.00	4.78			4.78
0293	10.55	6.00	10.00	1.17			1.17
0332	10.65	8.00	10.00	0.19			0.19
0341	10.48	4.00	10.00	3.15			3.15
0353	6.78	6.00	10.00	55.25			55.25
0368	28.41	22.00	24.00	3.55	8.19		11.74
0450	12.55	6.00	12.00	17.18			17.18
0459	12.66	8.00	12.00	3.44			3.44
0465	12.13	4.00	12.00	4.49			4.49
0467	12.13	8.00	12.00	0.43			0.43
0536	12.55	6.00	12.00	42.87			42.87
0571	8.08	8.00	12.00	3.04			3.04
0573	12.13	8.00	12.00	4.00			4.00
0575	12.13	8.00	12.00	2.46			2.46
0576	12.55	4.00	12.00	11.38			11.38
0580	12.55	6.00	12.00	7.01			7.01
0581	12.13	6.00	12.00	6.24			6.24
0651	14.66	8.00	14.00	6.17			6.17
0655	14.55	6.00	14.00	25.52			25.52
0658	NA	8.00	14.00	290.97			290.97
0761	14.37	8.00	14.00	2.73			2.73
0796	NA	8.00	14.00	91.14	14.23	39.68	145.05
0799	14.66	10.00	14.00	12.45			12.45
0887	16.55	8.00	16.00	37.18			37.18
0888	16.55	8.00	16.00	9.09			9.09
0907	16.13	8.00	16.00	2.31			2.31
0910	16.08	8.00	16.00	0.40			0.40
0940	16.55	8.00	16.00	14.61			14.61
0944	16.55	8.00	16.00	0.03			0.03
0953	16.55	8.00	16.00	57.02			57.02
0954	16.55	8.00	16.00	11.75			11.75
0984	18.55	8.00	18.00	22.14			22.14
1030	18.13	8.00	18.00	3.86			3.86
1032	18.08	8.00	18.00	1.92			1.92
1079	NA	8.00	18.00	263.48			263.48
1085	18.55	8.00	18.00	3.99			3.99
1173	20.13	6.00	20.00	21.33			21.33

1195	20.66	8.00	20.00	1.10			1.10
1206	20.78	8.00	20.00	15.83			15.83
1210	20.55	8.00	20.00	10.24			10.24
1268	20.66	8.00	20.00	0.30			0.30
1271	20.37	8.00	20.00	9.18			9.18
1273	20.55	8.00	20.00	17.73			17.73
1290	20.55	8.00	20.00	15.62			15.62
1302	20.37	10.00	20.00	0.50			0.50
1312	20.55	8.00	20.00	46.29			46.29
1318	20.51	10.00	20.00	4.61			4.61
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	NO GO	TOTAL
1336	20.55	8.00	20.00	13.17			13.17
1349	20.13	16.00	20.00	3.46			3.46
1495	NA	8.00	22.00	246.32	196.52		42.85
1497	22.55	10.00	22.00	49.53			49.53
1508	22.55	14.00	22.00	1.03			1.03
1519	24.13	4.00	24.00	0.54			0.54
1564	24.55	12.00	24.00	14.06			14.06
1619	24.55	8.00	24.00	4.80			4.80
1658	24.78	8.00	24.00	19.47			19.47
1670	28.78	8.00	24.00	14.63	52.40		67.03
1692	24.55	10.00	24.00	12.15			12.15
1721	24.66	10.00	24.00	0.29			0.29
1756	24.55	10.00	24.00	16.35			16.35
1800	24.55	8.00	24.00	48.59			48.59
1842	24.55	8.00	24.00	5.76			5.76
1907	30.63	12.00	26.00		160.61		160.61
1908	26.55	12.00	26.00	14.09			14.09
1909	26.55	8.00	26.00	15.33			15.33
1941	28.13	4.00	28.00	0.49			0.49
1943	28.63	8.00	28.00	43.56			43.56
2024	30.55	8.00	30.00	0.56			0.56
2032	30.45	16.00	30.00	2.09			2.09
2069	32.63	20.00	32.00	27.94			27.94
2130	34.13	4.00	34.00	0.21			0.21
2135	34.63	10.00	34.00	2.54			2.54
2154	34.78	8.00	34.00	75.92			75.92
2158	34.63	16.00	34.00	4.74			4.74
2184	34.13	26.00	34.00	0.37			0.37
2190	34.63	20.00	34.00	2.26			2.26
2201	36.63	8.00	36.00	1.73			1.73
2259	38.63	16.00	38.00	128.60			128.60
2267	38.63	24.00	38.00	28.60			28.60
2268	38.13	28.00	38.00	0.37			0.37
2339	40.63	14.00	40.00	4.38			4.38
2342	40.63	28.00	40.00	13.82			13.82
2354	40.63	10.00	40.00	0.46			0.46
2355	40.63	24.00	40.00	1.25			1.25
2361	42.63	8.00	42.00	9.64			9.64
2390	42.78	8.00	42.00	34.27			34.27
2409	42.63	8.00	42.00	17.37			17.37
2423	42.63	36.00	42.00	0.26			0.26
2484	44.63	18.00	44.00	122.48			122.48

2565	48.78	10.00	48.00	7.17			7.17
2625	48.64	44.00	48.00	3.95			3.95
2628	48.22	44.00	48.00	0.81			0.81
2637	50.40	8.00	50.00	0.55			0.55
2658	50.56	14.00	50.00	1.02			1.02
2661	50.63	8.00	50.00	8.19			8.19
2675	44.54	38.00	50.00	2.56			2.56
2690	50.63	42.00	50.00	0.85			0.85
2702	48.53	42.00	50.00	0.95			0.95
2705	50.59	44.00	50.00	0.27			0.27
2729	50.64	46.00	50.00	1.48			1.48
2739	50.24	46.00	50.00	8.98			8.98
2740	50.24	46.00	50.00	3.07			3.07
2742	50.24	44.00	50.00	2.04			2.04
2756	50.64	40.00	50.00	15.17			15.17
2760	50.64	44.00	50.00	3.04			3.04
2778	52.63	20.00	52.00	27.04			27.04
2802	52.64	46.00	52.00	0.96			0.96
2808	52.64	42.00	52.00	12.15			12.15
2813	52.64	44.00	52.00	8.72			8.72
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	NO GO	TOTAL
2817	52.22	48.00	52.00	1.04			1.04
2819	52.22	46.00	52.00	1.52			1.52
2848	52.64	48.00	52.00	1.71			1.71
2910	54.63	46.00	54.00	2.72			2.72
2915	54.24	48.00	54.00	4.20			4.20
2918	54.24	48.00	54.00	3.57			3.57
2923	54.64	48.00	54.00	14.52			14.52
2926	54.23	48.00	54.00	3.02			3.02
2965	54.64	52.00	54.00	3.03			3.03
3005	54.64	52.00	54.00	4.04			4.04
3021	54.24	54.00	54.00	0.51			0.51
3031	56.63	10.00	56.00	12.91			12.91
3047	56.59	50.00	56.00	37.54			37.54
3107	56.64	54.00	56.00	0.42			0.42
3119	56.24	50.00	56.00	3.84			3.84
3124	56.64	54.00	56.00	3.44			3.44
3153	56.64	50.00	56.00	27.50			27.50
3192	56.59	54.00	56.00	0.28			0.28
3201	58.64	52.00	58.00	47.89			47.89
3256	58.24	54.00	58.00	0.32			0.32
3260	58.64	48.00	58.00	9.39			9.39
3261	58.64	50.00	58.00	13.12			13.12
3265	58.63	20.00	58.00	26.99			26.99
3280	NA	54.00	58.00			3.16	3.16
3347	58.63	54.00	58.00	0.28			0.28
3394	60.63	52.00	60.00	2.46			2.46
3399	60.63	50.00	60.00	5.54			5.54
3567	60.24	54.00	60.00	1.27			1.27
3602	60.64	48.00	60.00	22.83			22.83
3603	60.64	54.00	60.00	4.18			4.18
3663	62.24	54.00	62.00	14.78			14.78
3664	62.24	54.00	62.00	6.28			6.28

3667	62.59	54.00	62.00	2.19			2.19
3670	62.24	54.00	62.00	16.09			16.09
3690	62.64	54.00	62.00	9.13			9.13
3701	62.64	54.00	62.00	3.31			3.31
3766	62.24	54.00	62.00	3.08			3.08
3789	64.64	52.00	64.00	6.02			6.02
3828	64.64	52.00	64.00	41.23			41.23
3832	64.22	54.00	64.00	30.14			30.14
3840	64.64	52.00	64.00	12.42			12.42
3849	64.64	52.00	64.00	139.78			139.78
3911	NA	52.00	64.00	73.66			73.66
3925	66.63	14.00	66.00	18.68			18.68
3927	66.64	54.00	66.00	4.07			4.07
3995	66.64	50.00	66.00	37.14			37.14
4006	68.64	54.00	68.00	15.23			15.23
4027	68.24	54.00	68.00	0.53			0.53
4158	70.64	52.00	70.00	33.86			33.86
4165	70.57	58.00	70.00	0.69			0.69
4166	70.57	56.00	70.00	0.84			0.84
4168	70.64	54.00	70.00	5.79			5.79
4169	70.64	54.00	70.00	4.87			4.87
4225	70.64	52.00	70.00	66.77			66.77
4231	72.24	54.00	72.00	2.45			2.45
4238	NA	52.00	72.00	97.48		134.73	232.21
4254	72.64	54.00	72.00	2.73			2.73
4267	72.64	8.00	72.00	0.55			0.55
4270	72.64	52.00	72.00	24.56			24.56
4277	72.64	54.00	72.00	26.35			26.35
4366	72.64	60.00	74.00	0.07			0.07
4388	74.63	64.00	74.00	8.34			8.34
	CLOSEDATE	ALD	RDD	ON TIME	LATE DELIV	NO GO	TOTAL
4418	76.64	52.00	76.00	25.50			25.50
4423	76.24	54.00	76.00	18.89			18.89
4430	76.64	56.00	76.00	4.80			4.80
4455	76.64	54.00	76.00	95.26			95.26
4470	76.73	56.00	76.00	8.45			8.45
4494	78.64	52.00	78.00	9.26			9.26
4497	78.64	54.00	78.00	4.01			4.01
4511	NA	54.00	78.00	54.68	36.48	3.24	94.41
4518	78.64	50.00	78.00	8.05			8.05
4522	78.64	8.00	80.00	0.55			0.55
4558	80.63	54.00	80.00	4.23			4.23
4564	82.64	54.00	82.00	0.60			0.60
4579	82.64	54.00	82.00	2.53			2.53
4724	84.55	50.00	84.00	3.14			3.14
4726	86.63	54.00	84.00	17.45	1.55		19.01
4732	84.64	58.00	84.00	3.55			3.55
4814	88.64	54.00	88.00	5.04			5.04
4887	90.64	54.00	90.00	0.76			0.76
5027	90.64	54.00	90.00	0.59			0.59
5045	90.64	54.00	90.00	88.77			88.77
5186	90.64	54.00	90.00	4.46			4.46
5206	90.64	54.00	90.00	26.85			26.85

5243	90.64	50.00	90.00	110.51			110.51
5286	94.64	54.00	90.00	50.71	154.69		205.39
5289	90.63	8.00	90.00	5.73			5.73
5299	90.64	44.00	90.00	25.53			25.53
5304	90.63	52.00	90.00	1.99			1.99
TOTAL				3896.14	624.68	180.82	4701.64

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